

# Performance analysis of null-steering beamformers with phase-only and amplitude-only control via BAT algorithm

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## Abstract

Recently, two types of BAT algorithm (BA)-based adaptive beamformers for pattern nulling in Uniformly Spaced Linear Arrays (ULA), utilizing either phase-only or amplitude-only control, have been proposed in previous studies. This paper evaluates the advantages and disadvantages of these approaches. Furthermore, to achieve higher performance, detailed recommendations for applying the optimized method in various scenarios are provided. To support this analysis, multiple ULA pattern scenarios with pre-set nulls were conducted. The simulation results are presented to validate the conclusions drawn.

**Keywords:** *Beamformer, Bat algorithm, Pattern nulling, Interference suppression, Null-steering, ULA antennas, Amplitude-only control, Phase-only control.*

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## 1. Introduction

Adaptive beamforming is a widely employed array signal processing technique in radar, sonar, and telecommunications. It enhances system performance by optimizing radio spectrum utilization, reducing interference, and minimizing power consumption. These techniques determine optimal weighting for antenna arrays, thereby achieving the desired radiation pattern [1].

The advancement of wireless communication and devices has led to significant electromagnetic interference in propagation environments. To address this challenge, smart antennas with null-steering capabilities have emerged as a promising solution.

Several control strategies have been extensively studied and implemented, including amplitude-only, phase-only, position-only, and complex weight methods, which regulate both amplitude and phase simultaneously [1-2]. Each approach presents distinct advantages and limitations.

Among these, the complex weights method is considered the most versatile and efficient. However, it is also the most complex and costly, as it requires individual controllers, phase shifters, and attenuators for each array element [3]. In contrast, the position-only method [4] employs a mechanical system to adjust the positions of array elements. While this approach modifies the system architecture, it also introduces considerable challenges in achieving precise control.

The amplitude-only control method [5-7] is relatively simple, as it involves adjusting only the amplitude of the excitation at each array element. Using this approach, a BA-based adaptive beamformer (AMP\_BA\_ABF) for adaptive null steering in ULA antenna patterns was proposed and successfully implemented in a previous study [8].

Pattern nulling based on phase-only control is less complex and particularly appealing for phased arrays [7, 9, 10]. In a previous study [11], a BA-based adaptive beamformer (PHA\_BA\_ABF) was proposed and successfully

implemented for pattern nulling in ULA antennas using the phase-only control method. Moreover, both proposed beamformers have been demonstrated to outperform PSO and GA techniques in interference suppression applications [8, 11].

BA is a contemporary evolutionary computation metaheuristic inspired by the echolocation behaviors of bats during prey detection, obstacle avoidance, and roost localization in low-light conditions. It has been successfully utilized to address a wide range of engineering challenges [12-17]. Comparative studies indicate that BA exhibits superior convergence speed, robustness, and precision relative to older metaheuristics such as PSO and GA [12]. Furthermore, BA has been effectively applied to beamforming, underscoring its potential as an advanced optimization tool for adaptive beamforming applications [8, 11, 22].

This paper evaluates the advantages and disadvantages of recent proposals [8, 11], providing appropriate recommendations for their application in various scenarios to achieve higher performance. The study has been conducted across four scenarios: operational speed, pattern nulling with a single null, multiple nulls, and broad nulls. The results are presented and explained in detail. Based on these findings, recommendations for selecting the most suitable approach for specific scenarios have been introduced to enhance interference suppression performance.

## 2. Fitness Function for Null Control

### 2.1. Array Factor of ULA

This study examines a uniform linear array (ULA) comprising  $2N$  isotropic elements, as illustrated in Figure 1. The array elements are symmetrically arranged relative to the center of the array, and the array factor is defined as follows [19]:

$$AF(\theta) = \sum_{n=-N}^N w_n e^{jndk \sin(\theta)} \quad (1)$$

where:  $w_n = w_n^{re} + jw_n^{im} = a_n e^{j\delta_n}$ : the complex excitation of  $n^{th}$  array element;  $k = \frac{2\pi}{\lambda}$ : the wave number; and  $\lambda$ : the wavelength;  $d$ : the distance between adjacent elements. The array factor can be expressed in terms of its real and imaginary components as follows:

$$AF(\theta) = \sum_{n=-N}^N w_n^{re} \cos(ndk \sin(\theta)) - w_n^{im} \sin(ndk \sin(\theta)) + j \left( \sum_{n=-N}^N w_n^{im} \cos(ndk \sin(\theta)) - w_n^{re} \sin(ndk \sin(\theta)) \right) \quad (2)$$

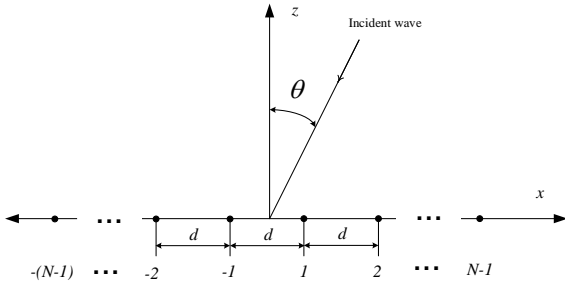


Figure 1: Geometry of 2N-element ULA.

## 2.2. Amplitude-only Control

In the previously proposed amplitude-only control-based null-steering beamformer [8],  $w_n^{im} = 0$  and  $w_{-n}^{re} = w_n^{re}$ . Therefore, from equations (1-3), the array factor can be rewritten as:

$$AF(\theta) = 2 \sum_{n=1}^N w_n^{re} \cos(ndk \sin(\theta)) \quad (3)$$

By applying this control to the ULA presented in Section 2.1, the number of attenuators can be reduced by half. However, this requires the addition of attenuators and amplitude controllers to the existing phased array system.

## 2.3. Phase-only Control

In the previously proposed phase-only control-based null-steering beamformer [11] and the study in [20],  $w_{-n}^{im} = -w_n^{im}$  and  $w_{-n}^{re} = w_n^{re}$ . Therefore, from equations (1-3), the array factor can be rewritten as:

$$AF(\theta) = 2 \sum_{n=1}^N w_n^{re} \cos(ndk \sin(\theta)) - w_n^{im} \sin(ndk \sin(\theta)) \quad (4)$$

According to this method, the number of phase shifters equals 2N. An important advantage is that this method can be applied to the existing phased array system without any additional cost.

## 2.4. Fitness Function

The Fitness function  $F$  has been built from [8, 11]:

$$F = \begin{cases} 1000 \sum_{i=1}^I [ |AF_0(\theta_i)|^2 ], & \text{for } \theta = \theta_i \\ \sum_{\theta=-90^\circ}^{90^\circ} [ |AF_0(\theta) - AF_d(\theta)|^2 ], & \text{elsewhere} \end{cases} \quad (5)$$

where:  $AF_0$  and  $AF_d$  corresponding to the optimized array factor obtained using BA and the desired array factor;  $\theta_i$  are the locations of the interference.

## 3. Proposed Beamformer

Amplitude-only or phase-only control-based beamformers using BA are developed in [8, 11], their flowcharts are presented in Figure 2.

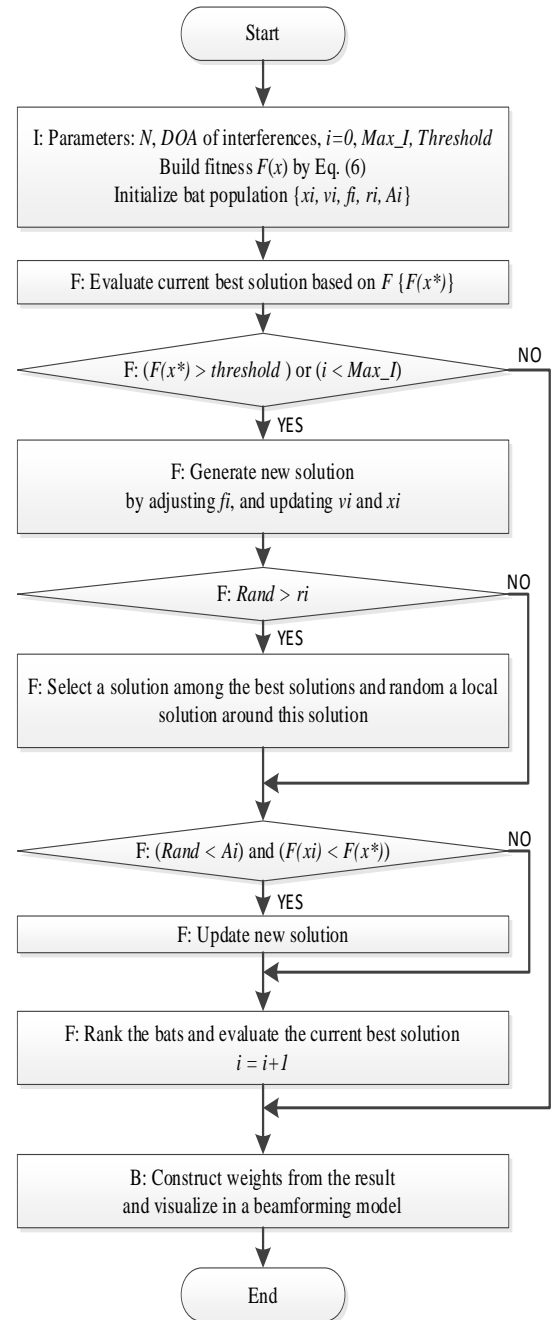


Figure 2: Flowchart of the proposed beamformers

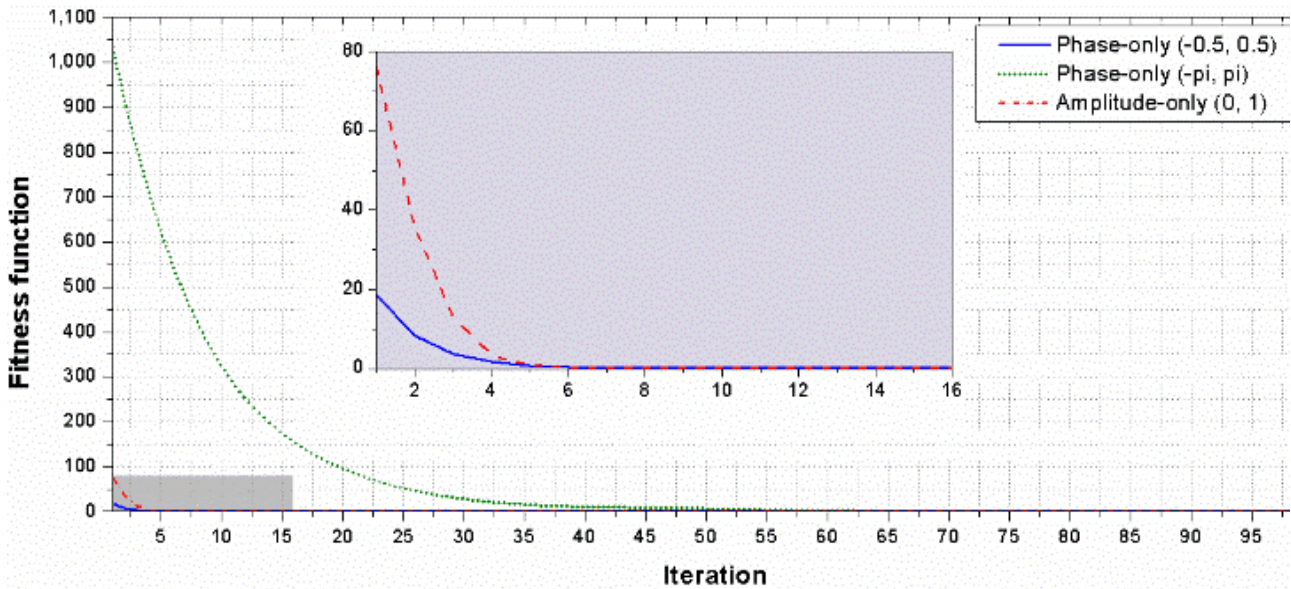


Figure 3: Fitness function of the proposals

The beamformer works as follows:

*Initializing (I):*

- Define the input parameters, including  $2N$ , the Direction of Arrival (DOA) of interferences; the current iteration count ( $i$ ); the maximum number of iterations ( $Max\_I$ ); and the termination criterion ( $Threshold$ ).
- Randomly initialize a population of bats, where each bat is characterized by its location ( $x_i$ ); velocity ( $v_i$ ); pulse frequency ( $f_i$ ); pulse rate ( $r_i$ ); and loudness ( $A_i$ ). Each bat represents a candidate solution.

*Finding the best solution (F)*

The beamforming algorithm iteratively computes and identifies the optimal solution. The process terminates when either the predefined  $Threshold$  or the maximum number of iterations ( $Max\_I$ ) is reached.

*Building array element weights (B)*

Based on the best solution, the beamformer determines the corresponding weights, which are then applied for pattern nulling the uniform linear array (ULA).

## 4. Numerical Results

To assess the effectiveness of the proposed approaches, four scenarios have been considered. The Chebyshev array weights are utilized to generate an optimal radiation pattern, ensuring a balance between the side lobe level (SLL) and the first-null beamwidth of the main beam for uniformly spaced arrays [21]. Accordingly, in this study, the array factor of a Chebyshev array with a SLL of -30 dB,  $d = \lambda/2$ , and  $2N = 20$  isotropic elements have been selected as the reference pattern to control both the SLL and the main beam's beamwidth.

The initial parameters of the Bat Algorithm for the investigated scenarios as follows: the step size of the random walk is 0.01; the boundary frequency values are set to  $f_{min}=0$  and  $f_{max}=1$ .

For AMP\_BA\_ABF [8]: the variable amplitude of the weights ranges from 0 to 1; the population size ( $pop$ ) is 100; and  $Max\_I = 1000$ , except to the first scenario in section 4.1.

For PHA\_BA\_ABF [11]: the variable phases of the weights are constrained within the range of -0.5 to 0.5 radians; the amplitudes of weights are derived from Chebyshev array weights;  $pop = 1000$ ; and  $Max\_I = 5$ , except for the first scenario in section 4.1.

### 4.1. Convergence Characteristic

In this scenario, the convergence ability of the proposed beamformers has been investigated for achieving the desired Chebyshev array pattern with a -30 dB SLL. To evaluate this, their convergence rates with a population of 100 and 100 iterations have been evaluated for side lobe suppression and illustrated in Figure 3. It can be seen that both the beamformers reach convergence after 5 iterations, with the objective function dropping below 1. However, Figure 3 and the numerical studies [8, 11] show that the amplitude-only control beamformer (*dash line*) converges faster, with an initial objective function value of approximately 76.3 compared to 18.5 for Phase-only control beamformer (*solid line*). This difference arises from the fact that AMP\_BA\_ABF utilizes the full amplitude range (0 to 1), while PHA\_BA\_ABF limits the phase range to  $[-0.5, 0.5]$  radians instead of the broader range of  $[-\pi, \pi]$  radians (dotted line). This phase range restriction results from the application of a small phase perturbation mode in Phase-only control, as discussed in studies [11, 22]. Despite the limited phase range, the controlled phase perturbation mode significantly improves the convergence speed of PHA\_BA\_ABF, as illustrated in Figure 3 by the solid and dotted lines.

### 4.2. Single Null

In the second scenario, optimized patterns featuring a single null have been demonstrated. This null can be positioned arbitrarily at any angle and, in this test case, has been placed at the peak of the second side lobe ( $14^\circ$ ). Figures 4 and 5 illustrate the optimized patterns with a single null, obtained using AMP\_BA\_ABF and PHA\_BA\_ABF, respectively. These patterns largely retain the characteristics of the original Chebyshev pattern, including a half-power beamwidth

(HPBW =  $7.64^\circ$ ) and a sidelobe level (SLL = -30 dB). However, in the certain sidelobes, the maximum SLL increases to -26 dB in the amplitude-only case and -24.24 dB in the phase-only case. All null depth levels (NDL) are lower than -90 dB.

As shown in Figure 4, a single symmetric null (two nulls at  $\pm 14^\circ$ ) has been correctly placed. This results from the symmetrical pattern obtained by AMP\_BA\_ABF through the main beam location ( $\theta = 0$ ) [8]. On the other hand, in Figure 5, with PHA\_BA\_ABF [11], a single null at  $14^\circ$  has been accurately set due to the anti-symmetrical pattern through the main beam direction ( $\theta = 0$ ) (solid line). Additionally, the pattern with a single symmetric null at  $\pm 14^\circ$  could not be achieved (dashed line).

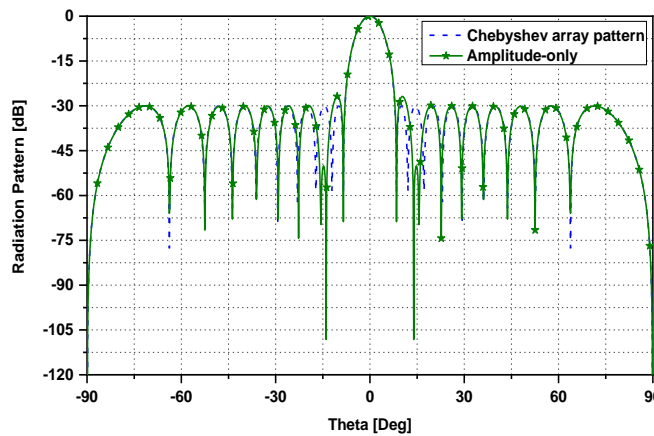


Figure 4: Amplitude-only: pattern with single symmetric null at  $14^\circ$

### 4.3. Multiple Nulls

In the third scenario, the proposed beamformers are used to separately set multiple nulls. Three symmetric nulls at  $14^\circ$ ,  $26^\circ$ , and  $33^\circ$  are generated using AMP\_BA\_ABF, as shown in Figure 6, while three nulls at  $-33^\circ$ ,  $-26^\circ$ , and  $14^\circ$  are produced using PHA\_BA\_ABF, as presented in Figure 7. These nulls correspond to the peaks of the second, fourth, and fifth sidelobes next to the main beam of the Chebyshev array pattern with a -30 dB SLL. As illustrated in Figures 6 and 7, the patterns with multiple nulls at the predefined locations have been accurately achieved. All null depth levels (NDLs) are deeper than -60 dB, all sidelobe levels (SLLs) are lower than -23 dB, and the half-power beamwidth (HPBW) remains approximately equal to that of the Chebyshev pattern. Once again, anti-symmetrical multiple nulls are imposed by PHA\_BA\_ABF, while multiple symmetrical nulls are placed by AMP\_BA\_ABF.

### 4.4. Broad Null

In interference suppression applications, a broad null is necessary when the direction of arrival of interferences fluctuates over time, is not precisely known, or requires continuous steering to maintain an optimal signal-to-noise ratio. To evaluate the effectiveness of broad interference suppression, the fourth scenario examines patterns with an imposed broad null at a predefined target. Specifically, a symmetrical broad null within the target sector  $[20^\circ, 50^\circ]$  has been generated using AMP\_BA\_ABF and is depicted in

Figure 8. Using PHA\_BA\_ABF, optimized patterns with broad nulls have been generated for the target sectors of  $[30^\circ, 40^\circ]$ ,  $[20^\circ, 50^\circ]$ ,  $[-50^\circ, -35^\circ]$ ,  $[-50^\circ, -40^\circ]$  and  $[20^\circ, 30^\circ]$ . These patterns are illustrated in Figures 9-11, respectively.

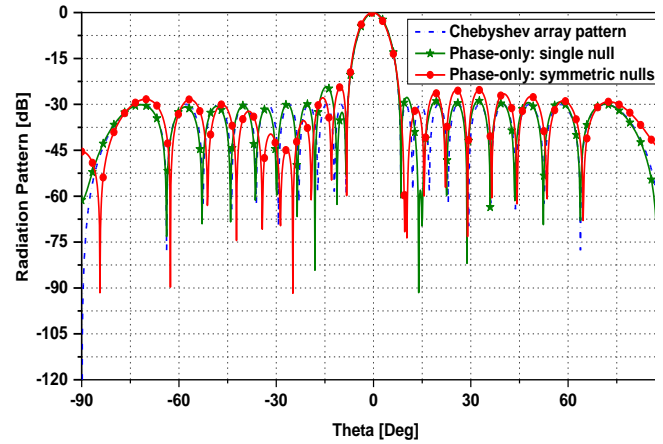


Figure 5: Phase-only: pattern with single null at  $14^\circ$  (solid line) and single symmetric null at  $\pm 14^\circ$

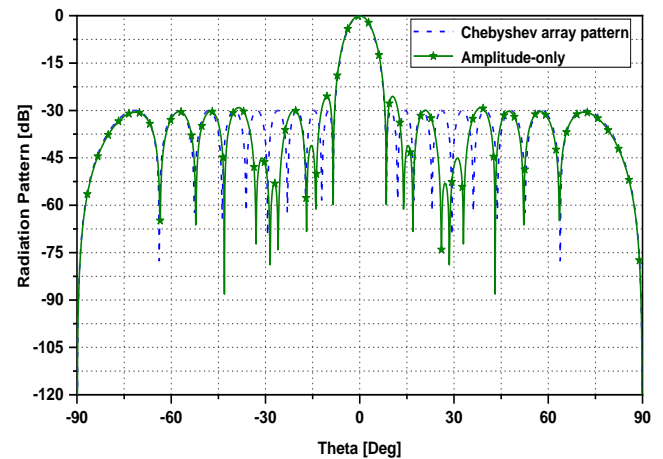


Figure 6: Amplitude-only: pattern with three symmetric nulls at  $14^\circ$ ,  $26^\circ$ , and  $33^\circ$

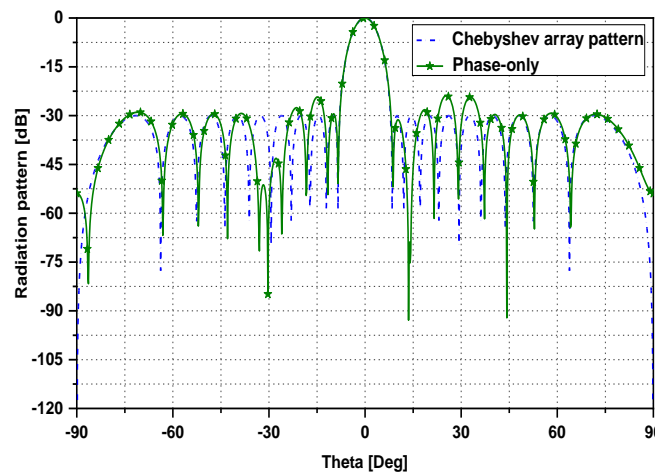


Figure 7: Phase-only: pattern with three nulls at  $-33^\circ$ ,  $-26^\circ$ , and  $14^\circ$

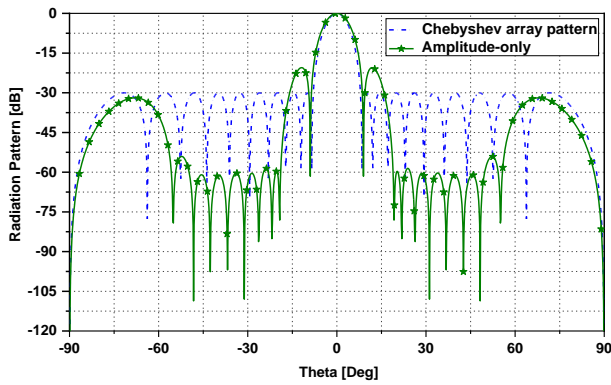


Figure 8: Amplitude-only: pattern with a symmetric broad null [20°, 50°]

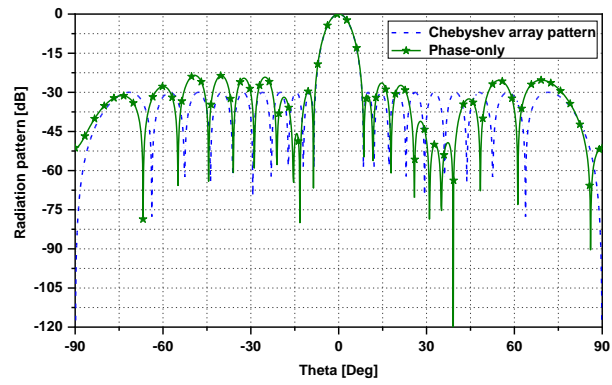


Figure 9: Phase-only: pattern with a broad nul [30°, 40°]

Table 1: Summarized evaluations of the proposals

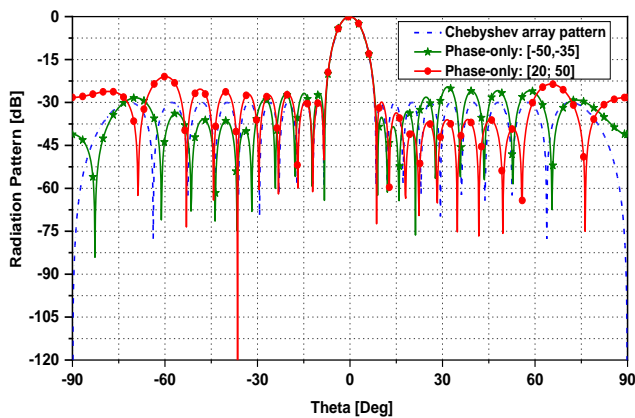
CHARACTERISTICS	AMP_BA_ABF	PHA_BA_ABF
Applied array	ULA: element-to-element distance: $\lambda/2$ ; 20 elements ( $2N$ )	ULA: element-to-element distance: $\lambda/2$ ; 20 elements ( $2N$ )
Optimization algorithm	BAT algorithm	BAT algorithm
Array elements	Ideal isotropic antenna	Ideal isotropic antenna
Array factor ( $AF(\theta)$ )	$AF(\theta)$ in (3)	$AF(\theta)$ in (4)
Fitness function ( $F$ )	$F$ in (5)	$F$ in (5)
Control parameters	Only amplitude of weights	Only phase of weights
Number of components	$N$ controllers and attenuators	$N$ controllers and $2N$ shifters
Hardware requirements for phased array systems	Add-in items	None
Software requirements for phased array systems	Add-in item using AMP_BA_ABF	Add-in item using PHA_BA_ABF
Convergence speed of the beamformers	$F < 0.1$ after 5 iterations with $pop$ is 100	Similar to that of Amplitude-only control in case of small phase perturbations mode
Interference suppression:		
General figures of nulling	Nulls must be symmetrical through $\theta$ of $0^\circ$	Anti-symmetrical nulls
• Single null and multiple nulls	Good performance: SLL and NDL	Good performance: SLL and NDL
• Broad null	Large width and high value of NDL	Small width and low value of NDL. The high value of NDL is still gained if the width is less than $10^\circ$
• Flexibility of null control	Be restricted to symmetrical constraints of the patterns symmetrical	Flexibility in desired locations of nulls
General recommendations	This proposal offers a straightforward implementation and is well-suited for the design of smart antenna systems that control only the amplitude of excitation for each array element. However, the approach has certain limitations: - The symmetrical pattern may introduce unintended nulls, leading to an increase in the level of certain side-lobes. - Additional costs may be incurred when integrating this method into existing phased array systems.	This proposal is adequate for most scenarios, even when implementing a broad null, provided its width does not exceed approximately $100^\circ$ . It offers a less complex and cost-effective solution for phased array systems, as the required controls are already available without additional expenses. However, the approach has certain limitations: - The inability to generate two nulls that are perfectly symmetrical around the main beam. - Difficulty in achieving large broad nulls.

Based on the simulation results in Figure 8 and in [8], a pattern with a symmetrical broad null is easily achieved with a deep null depth level (NDL), such as an NDL of approximately -60 dB in Figure 8. For example, the maximum NDL reaches approximately -50 dB for the broad null in the  $[30^\circ, 40^\circ]$  range (Figure 9), around -36 dB for the broad null in the  $[-50^\circ, -35^\circ]$  and  $[20^\circ, 50^\circ]$  ranges (Figure 10), and approximately -34 dB for the broad null in the  $[-50^\circ, -40^\circ]$  and  $[20^\circ, 30^\circ]$  ranges (Figure 11)

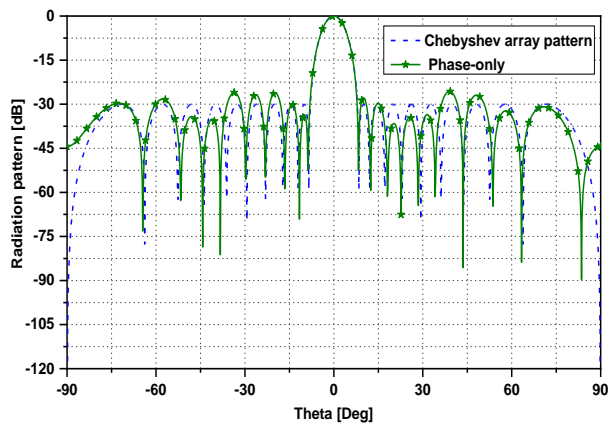
In the case of PHA\_BA\_ABF, numerical results from [8, 11] and Figures 9–11 show that both the width of the broad null and the NDL are smaller compared to AMP\_BA\_ABF. However, PHA\_BA\_ABF offers advantages over AMP\_BA\_ABF in terms of flexibility when setting the locations of broad nulls.

Further examples can be found in [8, 11]. A summarized evaluation of the results is presented in Table 1.





**Figure 10:** Phase-only: optimized patterns with a broad null at the sector of  $[-50^\circ, -35^\circ]$  (solid line) or  $[20^\circ, 50^\circ]$  (symbol line)



**Figure 11:** Phase-only: optimized patterns with two broad nulls at the sector of  $[-50^\circ, -40^\circ]$  and  $[20^\circ, 30^\circ]$

Complex weight control is introduced to leverage the benefits of both amplitude-only and phase-only control. However, since each array element requires a controller, a phase shifter, and an attenuator, this method is the most complex and costly. Nevertheless, it offers outstanding efficiency and flexibility [23].

## 5. Conclusion

In this study, recent proposals for interference suppression using Amplitude-only or Phase-only control have been evaluated for their benefits and limitations. To achieve this, four scenarios of ULA patterns with imposed preset nulls were conducted. The simulation results have been thoroughly demonstrated and analyzed to provide recommendations for optimal application in each scenario for improved performance.

For more realistic electromagnetic modeling concerning antennas and propagation, the effects of mutual coupling and element patterns will be investigated in future work.

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