Integral sliding mode control for nonlinear systems with prescribed settling time

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Abstract

This paper proposes a sliding mode controller design method using an integral sliding surface for a class of nonlinear systems. The method is developed to ensure that the system can effectively track the desired trajectory, with a settling time that can be flexibly adjusted according to the requirements of the actual system. The notable feature of the approach is the use of an integral sliding surface with parameters that are easily tuned, allowing control over the convergence rate of the system. As a result, the error between the actual output and the reference trajectory can be driven to zero within a desired time interval. The controller is designed not only to achieve high control performance but also to offer potential applications in control systems with stringent requirements on controlling the reaction speed. This method is particularly suitable for nonlinear systems with complex dynamic characteristics and strict settling time control requirements. The correctness of the theory and the effectiveness of the method are demonstrated through simulations conducted in Matlab.

Keywords: *Sliding Mode Control; Integral Sliding Surface; Nonlinear Systems; Prescribed Settling Time*

1. Introduction

Sliding Mode Control (SMC) has been extensively studied and applied in many fields of nonlinear control due to its robustness against disturbances and parameter variations of the system. Traditional studies mainly focus on the design of linear or nonlinear sliding surfaces to ensure stability, fast convergence speed, and to minimize the chattering phenomenon. Among them, the use of integral sliding surfaces has been proposed as a solution to eliminate steady-state error and improve the tracking performance of the system [1], [2], [3], [4], [5], [6], [7], [8], [9], [10], [11], [12], [13], [14], [15], [16], [17], [18]. In [1], [2], the authors proposed a dynamic integral sliding surface design method to reduce the chattering phenomenon commonly encountered in traditional sliding mode control. This method employs an integral sliding surface function with the ability to adaptively adjust parameters according to the nonlinear characteristics and disturbances of the system. Paper [3] proposes a technique for designing an integral sliding surface based on Sum of Squares (SOS) optimization, enabling the determination of sliding surface parameters to ensure system stability. The method also provides a sufficient condition for designing the sliding surface to guarantee system stability. In study [4], the author combines proportional and integral sliding surfaces to improve error elimination and enhance system stability. The sliding surface parameters are tuned based on the characteristics of a two-wheeled robot system, aiming to control the sliding velocity to reduce oscillations and mechanical wear. In [5], the author analyzes the role of sliding surface parameters in minimizing steady-state error and ensuring system stability, and also proves a sufficient condition for achieving stability. Studies [6], [7] select sliding surface parameters to ensure the system state converges to the origin and demonstrate that these parameters significantly affect both convergence time and control oscillation amplitude. This represents a noteworthy contribution in emphasizing the importance of adjusting sliding surface coefficients not only to ensure stability but also to enhance control performance. Sudhir Nadda [8] focused on the application of integral sliding surfaces in robot control, where high accuracy and the elimination of steady-state error are particularly important. Through simulations and experiments, the authors investigated the variation of sliding surface parameters and evaluated their effects on convergence speed and control signal oscillations. The results indicated that improper selection of coefficients could lead to overcontrol or undesired high-frequency oscillations, highlighting the necessity of optimal parameter design. The adaptive sliding mode control approach presented in [9] utilizes an adaptive update law to adjust the parameters of the integral sliding surface. Unlike approaches using fixed parameters, this method allows the coefficients to vary over time based on the system error and signal. Studies [10], [11], [12], [13], [14] further explored criteria for determining sliding surface parameters to ensure system stability and fast response. In paper [15] proposes an adaptive integral sliding mode control (ISMC) scheme that combines a tunnel prescribed performance function (TPPF) with RBF neural networks (RBFNN), enabling the vehicular platoon system to accurately track desired trajectories despite existing model uncertainties and external disturbances. The aforementioned studies have made significant contributions to the development and application of integral sliding surfaces in sliding mode control, particularly in analyzing the role and impact of

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sliding surface parameters on system stability and convergence speed. However, most of these works focus on ensuring asymptotic stability or general convergence, without providing a concrete method for selecting specific sliding surface parameters or establishing a quantitative relationship between these parameters and the desired settling time.

Unlike previous studies, this paper investigates the design of a sliding mode controller for state trajectory tracking that allows the settling time of the system to be adjusted through the simple selection of sliding surface parameters. This enables the designer to effectively and proactively control the system's response time in practical applications. The paper is outlined as follows. Section 2 presents the problem statement. Section 3 show design controller. Section 4 shows simulation result. Finally, Section 5 concludes the paper.

2. Problem statement

The nonlinear system is described by the state equation:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{B}(\mathbf{x})u \tag{1}$$

where $\mathbf{x} = [x_1, x_2, x_3, ..., x_n]^T$ is the state vector of the system, $u \in R$ is the control input, $\mathbf{f}(\mathbf{x})$ and $\mathbf{B}(\mathbf{x})$ are known functions, $\mathbf{f}(\mathbf{x}) = [f_1(\mathbf{x}), f_2(\mathbf{x}), ..., f_n(\mathbf{x})]^T$, $\mathbf{B}(\mathbf{x}) = [b_1(\mathbf{x}), b_2(\mathbf{x}), ..., b_n(\mathbf{x})]^T$ and $\sum_{i=1}^{n} |b_i(\mathbf{x})| \neq 0$. The control objective is to design the sliding mode controller such that the system state $\mathbf{x}(t)$ tracks the desired state trajectory $\mathbf{x}_d(t)$, with the tracking error defined as:

$$\mathbf{e} = \mathbf{x}(t) - \mathbf{x}_d(t) \tag{2}$$

The integral sliding surface is selected for system (1) as:

$$s = \mathbf{C}\mathbf{e} + \mathbf{C}_I \int_0^t \mathbf{e}(\tau)\mathbf{d}\tau \tag{3}$$

where $\mathbf{C} = [c_1, c_2, c_3, ..., c_n], \mathbf{C}_I = [c_1^I, c_2^I, c_3^I, ..., c_n^I]$ are coefficient matrices of the sliding surface. The integral sliding surface offers several significant advantages in nonlinear control, particularly for systems requiring precise tracking and the elimination of steady-state errors. The integral component ensures that the tracking error converges to zero when the system reaches a steady state, even in the presence of constant disturbances or model inaccuracies. In nonlinear systems affected by modeling uncertainties or unknown disturbances, the integral sliding surface enables a more effective compensatory control response by accumulating the tracking error, thereby helping to maintain the desired trajectory. Based on the prominent advantages of the integral sliding surface, we investigate the design of a trajectory tracking sliding mode controller with the capability to adjust the settling time through the tuning of sliding surface parameters. This approach holds practical engineering significance, as the settling time must be selected appropriately according to the inertia characteristics and specific requirements of each system. For systems with low inertia, such as small-scale robots or power electronic devices, a short settling time is needed due to the requirement for fast response. In contrast, for systems with large inertia or strict physical constraints such as ships, cranes, or high power steam turbines the settling time must be carefully considered. If the settling time is too short, the system may be forced to respond too quickly, potentially generating large impulses, strong oscillations, or control forces that exceed actuator limits, which could result in actuator damage or even severe impacts on the entire system. In response to these challenges, this paper focuses on the design methodology of a sliding mode controller using an integral sliding surface to address the aforementioned issues, which will be detailed in the following sections.

3. Design controller

To design the sliding mode controller, we divide the problem into two phases: the first phase is the reaching phase, during which the system states converge to the sliding surface; the second phase is the sliding phase, where the system slides along the surface toward the equilibrium point. This separation allows for a clearer and more systematic analysis, evaluation, and adjustment of each phase. Our proposed sliding mode control theorem is presented below, which explicitly specifies how to select the sliding surface parameters to ensure the system tracks the reference signal and achieves the desired settling time, in accordance with each application's specific requirements. This theorem facilitates the controller design process by ensuring the closed loop system's stability while enabling proactive control over the desired response time.

Theorem. *Consider the nonlinear system (1) and the integral sliding surface given by (3). If the control law is determined as:*

$$u = u_{eq} + u_s \tag{4}$$

where:

$$u_{eq} = -[\mathbf{CB}(\mathbf{x})]^{-1} (\mathbf{Cf}(\mathbf{x}) + \mathbf{C}_I\mathbf{x} - \mathbf{C}\dot{\mathbf{x}}_d - \mathbf{C}_I\mathbf{x}_d) \tag{5}$$

$$u_s = \begin{cases} -\eta \frac{s}{|s|+\gamma}, & \text{if } s \neq 0 \\ 0, & \text{if } s = 0 \end{cases} \tag{6}$$

$$\eta = \frac{\alpha V_2^p + \beta V_2^q + \xi}{|s|\ \mathbf{CB}(\mathbf{x})} \tag{7}$$

$$V_2 = 0.5s^2 \tag{8}$$

then the state vector of (1) will track the reference signal vector $\mathbf{x}_d$*, and the desired settling time* $T < T_{\max}$ *is achieved when the control parameters satisfy:*

$$0 < \lambda = \frac{c_k^I}{c_k} < \frac{1}{T_{me}},$$

$$T_s = \frac{1}{\alpha(1-p)} + \frac{1}{\beta(q-1)},$$

$$T_{\max} = T_{me} + T_s$$

in which: $\alpha, \beta > 0, 0 < p < 1, q > 1$*;* γ *and* ξ *are small positive coefficients; s is the integral sliding surface (3);* $\mathbf{C} = [c_k]$ *and* $\mathbf{C}_I = [c_k^I]$ *are the sliding surface parameter matrices;* c_k, $k = \overline{1,n}$ *are the adjustment parameters for the system to achieve the desired settling time* $T < T_{\max}$.

Proof:To prove the theorem, we consider the synthesis process of the trajectory-tracking sliding mode controller in two steps. First, we design the sliding surface to ensure its stability and guarantee that the sliding variables converge to zero within a finite time. Second, we synthesize the controller so that the system reaches and slides on the sliding surface, also within a finite time. Both steps jointly govern the settling time and ensure good tracking performance of the system. The detailed proof is presented below.
We set the intermediate variable:

$$\mathbf{z}=\int_0^t \mathbf{e}(\tau)\mathbf{d}\tau, \tag{9}$$

from (9), we have:

$$\dot{\mathbf{z}}=\mathbf{e}, \tag{10}$$

we rewrite (3) as:

$$s=\mathbf{C}\dot{\mathbf{z}}+\mathbf{C}_I\mathbf{z}=0. \tag{11}$$

Therefore, the design of parameters for the sliding surface (3) has been transformed into the problem of determining the parameter matrices $\mathbf{C}$, $\mathbf{C}_I$ such that the (11) is stable and all its state variables converge within a finite time interval. (11) can be rewritten as:

$$\sum_{k=1}^{n}\dot{z}_k=-\sum_{k=1}^{n}\frac{c_k^I}{c_k}z_k, \tag{12}$$

where $c_k, c_k^I, k=\overline{1,n}$ are elements of the matrices $\mathbf{C}, \mathbf{C}_I$. To determine the stability condition of equation (12), we select the following Lyapunov function:

$$V_1=\frac{1}{2}\sum_{k=1}^{n}z_k^2, \tag{13}$$

differentiating both sides of (12) yields:

$$\dot{V}_1=\sum_{k=1}^{n}z_k\dot{z}_k, \tag{14}$$

by substituting (12) into (14), we have:

$$\dot{V}_1=-\sum_{k=1}^{n}\frac{c_k^I}{c_k}z_k^2. \tag{15}$$

To ensure the stability of (12) if the Lyapunov derivative , from (15), we derive the condition for the stability of (12):

$$\lambda=\frac{c_k^I}{c_k}>0,\ k=\overline{1,n}, \tag{16}$$

choose $c_k^i>0$ and $c_k>0$. If $c_k, c_k^I, k=\overline{1,n}$ is chosen as in (16), then (12) is stable, at which point the variables of (12) converge to the origin $z_k\to 0$, meaning $e_k=x_k-x_k^d\to 0$ when the system slides on the sliding surface s. Subsequently, the paper determines the condition for selecting $c_k, c_k^I, k=\overline{1,n}$ such that the sliding variable's convergence rate is converge to zero within a finite time interval. From (12), we consider the component equation:

$$\dot{z}_k=-\frac{c_k^I}{c_k}z_k, \tag{17}$$

from equation (12), it can be observed that since the remaining equations share a similar structure and collectively contribute to the equation (12), for the aggregated equation to have all variables converge to zero simultaneously within the predetermined time interval T_{me}, we only need to determine the condition for equation (12) to ensure the variable z_k converges to zero within T_{me}.Subsequently, the parameters of the remaining equations will be selected such that their convergence time coincides with equation (17), thereby ensuring that the component equations of the aggregated equation (12) converge to within the finite time T_{me}. Equation (17) has the general solution:

$$z_k=z_k(0)e^{-\frac{c_k^I}{c_k}t}, \tag{18}$$

$z_k(0)$ is the initial condition at $t=0$, e is the Euler constant. From (10)and (18), we have:

$$e_k=\dot{z}_k=-\frac{c_k^I}{c_k}e^{-\frac{c_k^I}{c_k}T_{me}}, \tag{19}$$

From equation (19), in order to obtain the convergence rate of the sliding variable after a finite time T_{me},we have:

$$e_k=\dot{z}_k=-\lambda z_k(0)e^{-\lambda T_{me}}, \tag{20}$$

in control systems, to reduce the $e_k\to 0$ over a specified time period T_{me}, the attenuation criterion is selected as follows:

$$|e_k|\le\delta\,|z_k(0)|\,, \tag{21}$$

δ represents the ratio of the remaining error after the period of time T_{me}; for example, $\delta=0.01$ indicates that 1% remains compared to the initial value after a certain time T_{me}. The decay rate coefficient ensures a balance between the convergence speed and stability before the time [19],we select:

$$0<\delta\le\frac{1}{T_{me}e}; \tag{22}$$

in practice, to be feasible when controlling a real physical plant $T_{me}>1$. From (20)and (21), we have:

$$\left|-\lambda z_k(0)e^{-\lambda T_{me}}\right|\le\delta\,|z_k(0)|\to\lambda e^{-\lambda T_{me}}\le\delta \tag{23}$$

From (23), to design a control law ensuring the convergence rate of the sliding variable before a finite time T_{me},this requirement leads to the equation:

$$\lambda e^{-\lambda T_{me}}=\delta. \tag{24}$$

We consider the function:

$$f(\lambda)=\lambda e^{-\lambda T_{me}}, \tag{25}$$

the derivative of function (25) is:

$$f'(\lambda)=e^{-\lambda T_{me}}(1-\lambda T_{me}), \tag{26}$$

From (26), solving the equation $f'(\lambda)=0$ yields the solution:

$$\lambda=\frac{1}{T_{me}}, \tag{27}$$

It is easy to see that the function $f(\lambda)$ increases on the interval $\left(0,\frac{1}{T_{me}}\right)$ and and decreases on the interval $\left(\frac{1}{T_{me}},+\infty\right)$. It follows that $f(\lambda)$ attains its maximum at:

$$\lambda=\lambda^*=\frac{1}{T_{me}}, \tag{28}$$

$$f(\lambda^*) = \frac{1}{T_{me}e}. \tag{29}$$

From (28) and (29), we observe that for each value of δ in (22) there always exists a solution $0 < \lambda < \lambda^* \in (0, 1/T_{me})$ satisfying equation (22). On the other hand, within interval $\lambda \in (0, 1/T_{me})$,function (25) is monotonically increasing, meaning that if $\lambda_1 < \lambda_2$, then $f(\lambda_1) = \delta_1 < f(\lambda_2) = \delta_2$. Therefore, to ensure the sliding variable's convergence rate meets the practical requirements of the system within a finite time interval smaller than T_{me}, we only need to select an appropriate value of λ belonging to interval $\lambda \in (0, 1/T_{me})$. In summary, the range for selecting λ to ensure the convergence time of the sliding variables is less than T_{me} is:

$$0 < \lambda < \frac{1}{T_{me}}. \tag{30}$$

From (16) and (30), we derive the method for selecting the sliding surface parameters to force all sliding variables of the sliding surface (3) to stabilize within a predetermined time interval T_{me} as:

$$0 < \lambda = \frac{c_k^I}{c_k} < \frac{1}{T_{me}}, c_k^I > 0, c_k > 0, k = \overline{1,n} \tag{31}$$

Next, the sliding-mode controller is designed for trajectory tracking of system (1) using the sliding surface defined in (3). The parameters of the sliding surface are chosen according to (31), and the controller takes the following form:

$$u = u_{eq} + u_s, \tag{32}$$

in which u_s is the component that drives the system toward the sliding surface (3), and u_{eq} is the control component that keeps the system on the sliding surface. The component u_{eq} is determined from the condition $\dot{s} = 0$:

$$\dot{s} = \mathbf{C}\dot{\mathbf{e}} + \mathbf{C}_I\mathbf{e}, \tag{33}$$

substituting (2) into (33), we obtain:

$$\dot{s} = \mathbf{C}(\dot{\mathbf{x}} - \dot{\mathbf{x}}_d) + \mathbf{C}_I(\mathbf{x} - \mathbf{x}_d) = 0, \tag{34}$$

next, substituting (1) into (34), we obtain:

$$\mathbf{C}(\mathbf{f}(\mathbf{x}) + \mathbf{B}(\mathbf{x})u_{eq} - \dot{\mathbf{x}}_d) + \mathbf{C}_I(\mathbf{x} - \mathbf{x}_d) = 0 \tag{35}$$

From (35), we obtain :

$$u_{eq} = -[\mathbf{CB}(\mathbf{x})]^{-1}(\mathbf{Cf}(\mathbf{x}) + \mathbf{C}_I\mathbf{x} - \mathbf{C}\dot{\mathbf{x}}_d - \mathbf{C}_I\mathbf{x}_d). \tag{36}$$

Next, we determine u_s based on the sliding mode existence condition. We choose the Lyapunov function:

$$V_2 = 0.5s^2, \tag{37}$$

Taking the derivative of both sides of the Lyapunov function:

$$\dot{V}_2 = s\dot{s} \tag{38}$$

Substituting (3) into (38), we obtain:

$$\dot{V}_2 = s(\mathbf{C}\dot{\mathbf{e}} + \mathbf{C}_I\mathbf{e}) \tag{39}$$

Next, substituting (1),(2) and (32) into (39), we obtain:

$$\begin{aligned}\dot{V}_2 =& s(\mathbf{C}(\mathbf{f}(\mathbf{x}) + \mathbf{B}(\mathbf{x})u_{eq} - \dot{\mathbf{x}}_d) + \mathbf{C}_I(\mathbf{x} - \mathbf{x}_d)) \\ &+ s(\mathbf{CB}(\mathbf{x}))u_s.\end{aligned} \tag{40}$$

By combining equation (35), expression (40) becomes:

$$\dot{V}_2 = s(\mathbf{CB}(\mathbf{x}))u_s. \tag{41}$$

In order for the sliding function s to converge to the sliding surface within a time interval $t < T_s$, based on the stability criteria established in the finite-time stability results presented in [20], from (41) we select:

$$\dot{V}_2 = s(\mathbf{CB}(\mathbf{x}))u_s \le -\alpha V_2^p - \beta V_2^q, \tag{42}$$

$\alpha, \beta > 0, 0 < p < 1, q > 1$. T_s from [20]:

$$T_s = \frac{1}{\alpha(1-p)} + \frac{1}{\beta(q-1)}. \tag{43}$$

We design the controller such that condition (42) is always satisfied :

$$u_s = -\eta\,\mathrm{sgn}(s), \tag{44}$$

where:

$$\eta = \frac{\alpha V_2^p + \beta V_2^q + \xi}{|s|\,\mathbf{CB}(\mathbf{x})}, \tag{45}$$

ξ is a small positive constant. When the system has not yet reached the sliding surface u_s in (44) will drive the system toward the sliding surface. When the system reaches the sliding surface, u_{eq} in (36) will maintain the system on the sliding surface. To reduce the chattering effect, an approximate replacement function is used [21]:

$$\mathrm{sgn}(s) \approx s/(|s| + \gamma), \tag{46}$$

γ a small positive constant. The settling time of the system is governed by T_{eq} and T_s whereby the system will settle within a finite time interval $T < T_{\max}$:

$$T_{\max} = T_s + T_{me}. \tag{47}$$

From expressions (31), (44), (36) and (47), it can be observed that to adjust the desired settling time T, with $T < T_{\max}$, we only need to tune the sliding surface parameter $\mathbf{C} = [c_k], k = \overline{1,n}$. Furthermore, our method allows for automatic updating of the sliding condition (45) without requiring manual intervention whenever T is modified, thereby simplifying and enhancing the efficiency of the sliding mode controller design process. Moreover, when the system is subject to disturbances or modeling errors, the sliding mode is always guaranteed. Finally, expressions (31), (32), (36), (43) , (44), (45), (46) and (47) lead to the statement of the theorem;

The theorem is proved.

Next, we illustrate by simulating a specific example to demonstrate the correctness and effectiveness of the proposed theorem.

4. Simulation results

The class of nonlinear systems used for simulation is described by the state equation:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}x_2\\ -1.5x_1 - 2x_2 - 0.15x_1^2\end{bmatrix} + \begin{bmatrix}0\\ 1 + x_1^2\end{bmatrix}u \tag{48}$$

The controller is designed according to the theorem: (4),(5),(6),(7),(8) ; The parameters follow from the theorem:
$T_s = 4 \rightarrow p = 0.5, q = 2, \alpha = 1, \beta = 0.5$;
$T_{eq} = 4 \rightarrow \lambda \in (0, 1/4)$ choose $\lambda = 0.0013$;
Reference signal:
$y_d = 1, \dot{y}_d = 0 \rightarrow \mathbf{x}_d = \left[x_1^d, x_2^d\right]^T = [1, 0]^T$; Desired settling time: $T < T_{\max} = T_s + T_{eq} = 8s$. In the simulation of case 1, the sliding surface parameter is chosen as : $\mathbf{C} = [1, 0.15]$, $\mathbf{C}_I = \lambda \mathbf{C}$.
Simulation results on MATLAB software [Fig 1–Fig 6]:

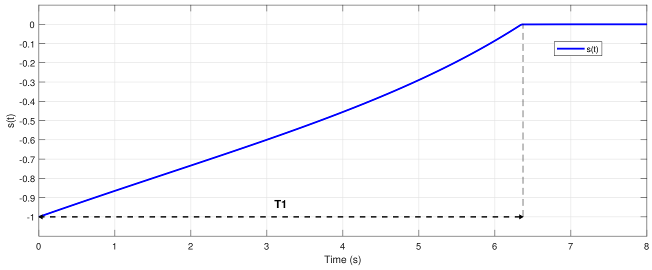

Figure 1: Integral sliding surface s(t).

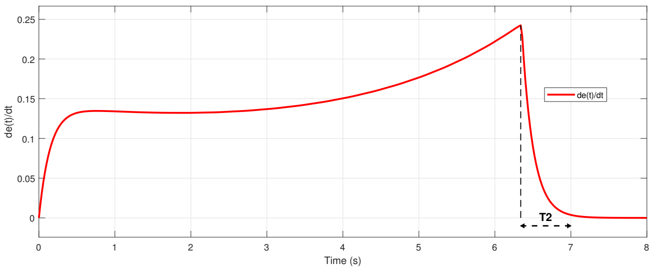

Figure 2: Velocity e(t)

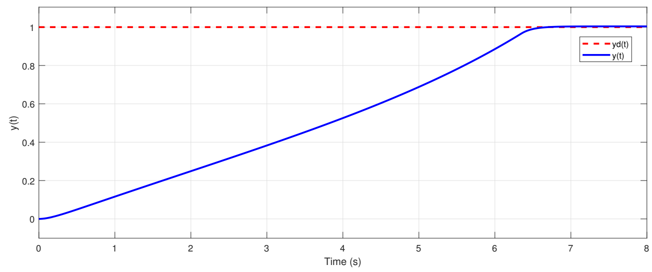

Figure 3: y(t) tracking yd in finite time.

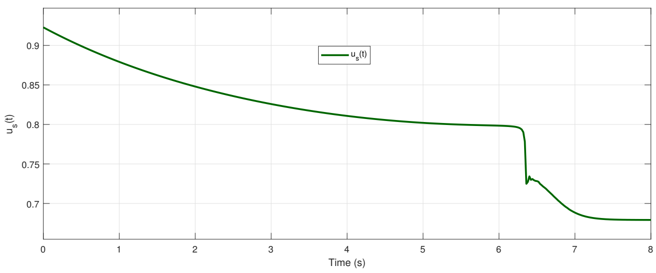

Figure 4: Control signal us(t).

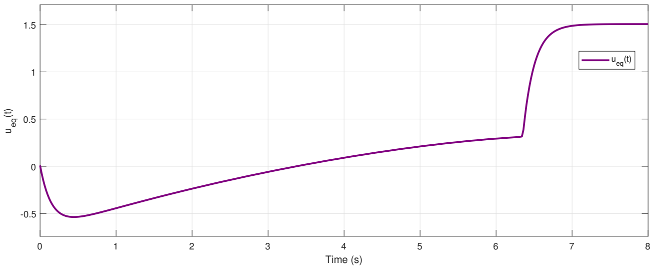

Figure 5: Control signal ueq(t).

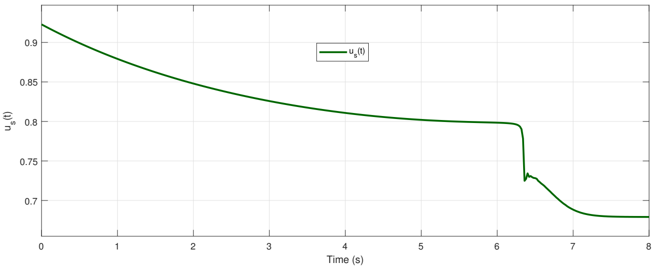

Figure 6: Control signal u(t) = ueq(t)+us(t).

The simulation of Case 2 demonstrates that the settling time can be reduced by properly tuning the sliding surface parameters, $\mathbf{C} = [1, 0.6]$, $\mathbf{C}_I = \lambda \mathbf{C}$.
Simulation results on MATLAB software [Fig 7–Fig 12]:

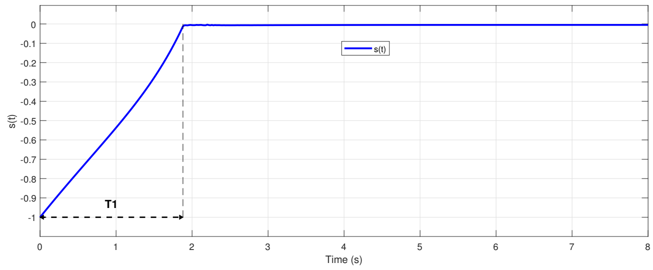

Figure 7: Integral sliding surface s(t).

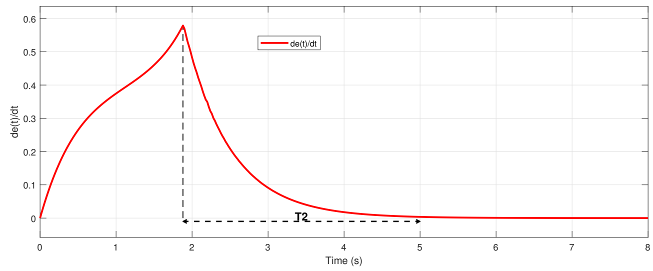

Figure 8: Velocity e(t)

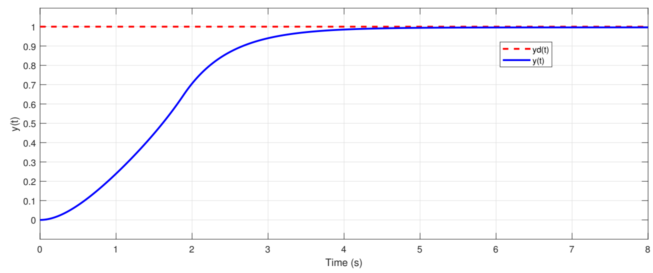

Figure 9: y(t) tracking yd in finite time.

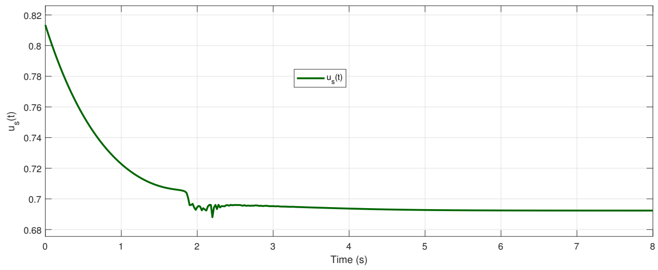

Figure 10: Control signal us(t).

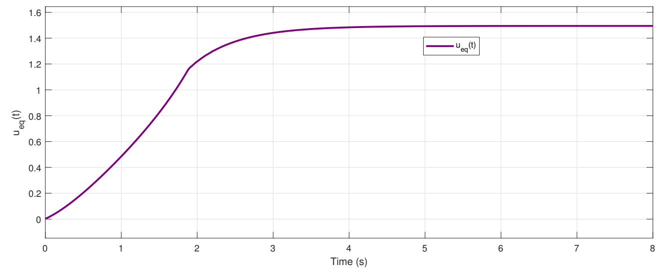

Figure 11: Control signal ueq(t).

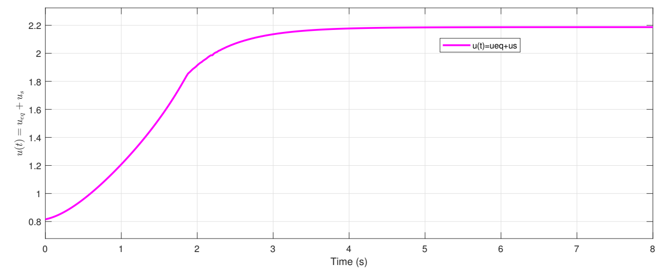

Figure 12: Control signal u(t) = ueq(t)+us(t).

The simulation results for the two investigated cases show that the proposed controller achieves good tracking performance, as illustrated in [Fig 3;Fig 9], with control signals in [Fig 4,Fig 5,Fig 6;Fig 10,Fig 11,Fig 12]. The desired settling time is ensured with $T = T_1 + T_2 < T_{\max} = 8$s as shown in [Fig 1,Fig 2;Fig 7,Fig 8]. Notably, the adjustment of the settling time can be performed in a technically simple and effective manner, solely by modifying the sliding surface parameters **C**. The simulation results are fully consistent with the theoretical analyses presented. This indicates that the proposed controller not only ensures superior control quality but also offers high flexibility in adjusting the settling time, with a simple and easy to implement design procedure.

5. Conclusion

In this paper, the sliding mode control design method employing an integral sliding surface is proposed for a class of nonlinear systems to ensure trajectory tracking with a finite settling time. A notable contribution of the study is the introduction of a new theorem for designing a reference trajectory tracking controller, which enables the adjustment of the settling time through a simple and effective selection of the sliding surface parameters. The theorem is rigorously proven, providing a solid theoretical foundation for the stability and convergence of the system within a finite time interval. Simulation results in Matlab demonstrate the effectiveness and practical applicability of the proposed method. This approach exhibits the capability to adjust the response speed merely by tuning the sliding surface parameters, while ensuring that the tracking error between the output and the reference trajectory gradually converges to zero within the desired time. Therefore, the proposed method holds strong potential for broad application in control systems requiring high precision and explicit control of response time.

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